

# Break Even – Fixed and Variable Expenses

**SUBMITTED BY:** Nina Hoe, University of Pennsylvania

**SUBJECT(S):** Computation

**GRADE LEVEL(S):** 9, 10, 11, 12

## ≡ OVERVIEW:

If done as an isolated lesson, students are introduced to the idea of break even, revenues and expenses (both fixed and variable). The lesson begins with a whole group discussion of these topics, applied to different business scenarios. (If this lesson is done in sequence with the first break-even lesson – fixed expenses only – this can be a short review.) Students can share their answers from the previous lesson. Students are asked to think about their relationship between fixed and variable expenses. In small groups, students work through calculating break-even points for businesses with fixed and variable expenses. Finally, students report on their findings and discuss the importance of calculating break-even point and further discuss the relationship between the two.

## ≡ RELATED ARTICLES:

- [“The Silicon Valley Start-up that Began with Legos and a Market Need”](#)
- [“Research Strategies for New Investors”](#)

## Standards:

### NBEA Standard(s):

- Number Relationships
- Mathematical Foundations
- Patterns, Functions and Algebra

## Common Core Standard(s):

### Modeling

F-LE.2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

A-CED.1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

## Objectives/Purposes:

- Understand the concept of breaking even and the relationship between revenues and expenses.
- Understand the relationship between fixed and variable expenses in a break-even mode.
- Model break-even situations graphically.
- Solve systems of equations numerically and graphically.

## Knowledge@Wharton Articles:

- [“Power, Leverage and a Strategic Approach to the Negotiation Process”](#)
- [“Smooth as Silk: Product Diversification Gives Afghan Women a Competitive Edge”](#)
- [“Chinese Firms Excel in ‘The Art of Price War’”](#)

## Whole Class Discussion: (10 mins if not discussed in Computation 30)

1. If you were to start an Internet-based company, what factors would be important to you?
2. What do you think it means to “break even”? Have you heard this term before?

Play the Wharton Global Youth Program (WGYP) Glossary: **Break Even**

*“Break even is when revenues and expenses are equal. Or, put another way, when net income is zero. The Bluth Company is projected to break even, at selling 100,000 tables. If they sell less tables, they’ll make a loss. If they sell more tables, they’ll make a profit.”*

Play the WGYG Glossary: [Break Even Point](#)

*“Break even point refers to either the quantity of output sold, or the total revenues where operating income is zero. The Bluth Company’s break even point in quantity is 100,000 tables, and in sales dollars, is \$1 million.”*

Essentially, the break-even point is when:

$$\text{Revenues} = \text{Expenses}$$

$$\text{Revenues} - \text{Expenses} = 0 \text{ (or the Net Income} = 0)$$

3. What are revenues?

Play the WGYG Glossary: [Revenues](#)

*“Revenues are the sales of products, merchandise and services that a company makes to customers due to the normal business activities. The Philadelphia Phillies, the 2008 World Series Champions, generated a record-breaking \$216 million in revenues from ticket sales, merchandise, and refreshments last year.”*

4. What is the difference between revenues and profits?

5. What are expenses?

6. What are fixed vs. variable expenses/costs?

Play the WGYG Glossary: [Fixed/Variable Cost](#)

*“A fixed cost is a cost that remains unchanged in total, regardless of changes in the level of total activity or volume. Earthquake insurance for a table factory is a fixed cost, because regardless of whether 200 or 2000 tables are made, the cost for earthquake insurance will be the same. A variable cost is a cost that changes directly in proportion to changes in the level of total activity, or volume. The wood used to make a table is a variable cost, because each additional table requires additional wood for the tables to be made.”*

7. If you ran a corner store – what would your **revenues** be? (***your total sales, how much money was in the register at the end of each day***)

8. If you ran a corner store – what would your **fixed expenses** be? (*paying employees, rent for the space or mortgage, utilities, etc*)
9. If you ran a corner store – what would your **variable expenses** be? (*cost of food and merchandise to stock the shelves, potentially adding additional employees if the need got high enough*)
10. Which do you think is better, a business with more variable expenses or less? More fixed expenses or less? What do you think is the idea relationship between the two?

Different businesses have different profiles and different relationships between and proportions of fixed and variable.

11. Brainstorm some businesses that would have higher fixed costs and lower variable costs. (*ex: barber shop, massage parlor, bicycle rental shop*)
12. Brainstorm some businesses that would have lower fixed costs and higher variable costs. (*ex: grocery or retail store, anything where you are selling something tangible*)
13. Why is knowing your break-even point important?
14. What does it mean if revenues are greater than expenses?
15. What does it mean if revenues are less than expenses?

Read the Knowledge@Wharton article: [“Power, Leverage and a Strategic Approach to the Negotiation Process”](#)

16. Why is it important for these women to know their break-even points?

Every start-up business has to anticipate that in the beginning, the expenses will be larger than the revenues and that the business will initially be losing money. Business owners and investors look to break-even projections to analyze the risk involved with starting a particular business. Companies think about break-even points in different ways, and ask different questions regarding breaking even. Sometimes, a new company will expect that expenses will exceed revenues in the beginning, so they may ask – given this **pricing** strategy, at what point in time will we reach a break-even point? Another company may use the break-even point as a goal, and ask – if we want to break even within three years, what pricing strategy must we use?

Play the WGYG Glossary: [Pricing](#)

*“Pricing is the price a firm sets for the products it sells. Wal-Mart has a pricing policy to be as low or lower than any of the competitors. Wal-Mart will match or beat anyone’s prices.”*

There are also initial, start-up, or capital costs or investments involved in starting a business that may not be figured into the monthly expenses. For example, to start up a corner store, you would need to invest in shelving, a price gun, signage, etc.

Small Group/Pair Activity:

[Student Worksheet](#)

**FIXED AND VARIABLE EXPENSES (20 – 30 minutes) (It might be best to do this out loud with the class.)**

You want to open an ice cream parlor/store.

1. Based on the previous discussion, is this business likely to have higher fixed costs or variable costs? Why? **(Higher variable costs. The more ice cream sold, the greater the expenses since you have to supply the ice cream.)**
2. What are some of the fixed expenses associated with this business? **(rent, utilities, equipment rentals)**
3. What are some of the variable expenses associated with this business? **(ice cream, cones, cups, spoons, napkins)**
4. If one person comes in to buy ice cream or 100 people come in to buy ice cream, how does this affect your expenses in general? **(Much more ice cream needs to be bought.)**
5. What would be some of your initial, or start-up costs? **(freezers, signage, etc.)**

You have determined that the expenses for this business are as follows:

Fixed Expenses	Monthly Cost	Variable Expenses	Cost per sale/each
Store rental	\$930	Ice cream	\$0.50
Utilities	\$400	Cones/Cups	\$0.15
Payroll: 2 employees – \$8.50/hour – 40 hours per week	\$2,720	Spoons	\$0.05

Insurance	\$100	Toppings	\$0.13
Advertising/Marketing	\$100	Napkins	\$0.02
<b>Total</b>	<b>\$4,250.00</b>	<b>Total</b>	<b>\$0.85</b>

6. Each ice cream that you sell comes with a fixed portion of ice cream, a cup or cone, a spoon, a topping of choice, and a napkin. Explain the cost per sale/each. **(These are variable costs that total up to the cost per sale for the shop owner.)**
7. Write an equation that models your monthly expenses as a function of the number of ice creams you sell, incorporating both your fixed and variable expenses. Let  $y$  = total monthly expenses (\$) and  $x$  = # of ice creams you sell per month. **( $y = .85x + 4250$ )**
8. As the number of ice creams you sell per month increases, how does this affect your monthly expenses? **(monthly expenses increase by \$0.85 for each ice cream sold)**
9. You decide to sell your product for \$3.35. Write an equation that models your total monthly revenues as a function of the number of ice creams you sell. Let  $y$  = total monthly revenues (\$) and  $x$  = # of ice creams you sell per month. **( $y = 3.35x$ )**
10. As the number of ice creams you sell per month increases, how does this affect your monthly revenues? **(They increase by \$3.35 per sale.)**
11. What is the starting point of the monthly expenses (when you have sold 0 ice creams)? **(\$4,250)**
12. What is the starting point of your monthly revenues (when you have sold 0 ice creams)? **(\$0)**
13. Which increases at a faster rate, your expenses or revenues? How do you know? **(revenues – slope = 3.35 vs. 0.85 for expenses)**
14. How many ice creams do you need to sell per month to break even? (When do the lines meet?) Solve this numerically. **(1,700)**
15. If you are open 7 days a week for a total of 40 hours per week (and there are approximately 4 weeks per month), approximately how many ice creams is this per day? **(60)**
16. Your initial investment costs for this business including the freezers, scoops, cash register, and signage totaled \$7,000. If you believe that you can sell 2,100 ice creams per month for \$3.50 each, estimate how long it will take you to break even accounting for this initial investment. **(Answers will vary. Expenses:  $y = 4250x + 7000$ ; Revenues:  $y = (3.5 \times 2100)x$ ,  $y = 7350x$  à 2.26 months)**
17. What are your monthly revenues now? **(\$7,350)**
18. Write an equation that models cumulative total revenues as a function of the time (in months). Let  $y$  = revenues (\$) and  $x$  = # of months. **( $y = (3.5 \times 2100)x$ ,  $y = 7350x$ )**
19. Write an equation that models cumulative total expenses, accounting for both initial and fixed expenses, as a function of time (in months). Let  $y$  = expenses (\$) and  $x$  = # of

months.  $(y = (0.85 \times 2100)x + 4250x + 7000) = 1785x + 4350x + 7000 = 6035x + 7000$

20. At what point do these lines meet? Solve this numerically and graphically. **(5.3 months)**

### **Tying It All Together:**

#### Whole Class Discussion: (10 mins)

1. Have students report back on their findings. Student groups can be responsible for putting different # problems on the board.
2. Why is it important to calculate your break-even point when starting or planning a business?
3. As a business owner, what is the relationship between fixed and variable expenses?
4. How do you balance these to maximize your profits?
5. Would you rather have a business with more fixed or variable expenses?

**Practice Outside of the Classroom:** Look at other businesses and think about what the fixed and variable expenses are. Think about where their break-even points might be.

### **What worked and What I Would Do Differently:**

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